1D) $\displaystyle\int \frac{3x}{\sqrt{4x^2+5}}\,dx$

$\displaystyle\int \frac{dx}{x}$

$\displaystyle\int \left(\frac{1}{x}\right)dx$

let $u = 4x^2+5$

$du = 8x\,dx$

$\frac{1}{8}du = x\,dx$

$\Rightarrow \displaystyle\frac{1}{8}\int 3\left(\frac{1}{\sqrt{u}}\right)du = \frac{3}{8}\int u^{-1/2}\,du$

$= \displaystyle\frac{3}{8}\left(\frac{u^{1/2}}{1/2}\right) + C$

$= \displaystyle\frac{6}{8}\sqrt{4x^2+5} + C$

6A)

$$\int \frac{x^2}{1+x^6}\,dx \qquad \frac{1}{3}\int \frac{1}{1+u^2}\,du$$

$x^6=(x^3)^2$

Let $u=x^3$

$du=3x^2\,dx$

$\frac{1}{3}du=x^2\,dx$

$$=\frac{1}{3}\tan^{-1}(u)+C$$

$$=\frac{1}{3}\tan^{-1}(x^3)+C$$

6D $\displaystyle\int \frac{dx}{\sqrt{x}(1+x)} \Rightarrow 2\int \frac{1}{1+u^2}\,du$

Let $u = \sqrt{x}$ $\quad 1+x = 1+(\sqrt{x})^2$

$du = \frac{1}{2\sqrt{x}}\,dx$ $\quad 2\tan^{-1}(u) + C$

$2\,du = \frac{1}{\sqrt{x}}\,dx$ $\quad 2\tan^{-1}(\sqrt{x}) + C$

5D)

$$\int \frac{e^x}{1+e^x}\,dx \Rightarrow \int \frac{1}{u}\,du$$

Let $u = 1+e^x$

$du = e^x dx$

$= \ln(u)+C$

$= \ln(1+e^x)+C$

$u = 1+e^x$

$u-1 = e^x$

5c)

$$\int \frac{\sin 3\theta}{1+\cos 3\theta}\,d\theta \Rightarrow -\frac{1}{3}\int \frac{1}{u}\,du$$

Let $u = 1+\cos 3\theta$

$du = -3\sin 3\theta \, d\theta$

$-\frac{1}{3}du = \sin 3\theta \, d\theta$

$= -\frac{1}{3}\ln(u) + C$

$= -\frac{1}{3}\ln(1+\cos(3\theta)) + C$

$$\int \frac{\ln(\ln x)}{x}\,dx \Rightarrow \int \ln(u)\,du$$

Let $u = \ln x$

$du = \frac{1}{x}dx$

6.3/7-27

6a) $\frac{6}{3}=3$

$$\int \frac{x^2}{1+x^6}\,dx \Rightarrow \frac{1}{3}\int \frac{1}{1+u^2}\,du$$

$1+x^6$

$=1+(x^3)^2$

Let $u=x^3$

$du=3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$$= \frac{1}{3}\tan^{-1}(u) + C$$

$$= \frac{1}{3}\tan^{-1}(x^3) + C$$

4a)

$$\int x^2\sqrt{1+x}\,dx \Rightarrow \int (u-1)^2\sqrt{u}\,du$$

Let $u = 1+x$;

$du = dx$

$u-1 = x$

$$= \int (u^2-2u+1)\,u^{1/2}\,du$$

$$= \int u^{5/2}-2u^{3/2}+u^{1/2}\,du$$

$$= \frac{2u^{7/2}}{7}-\frac{4u^{5/2}}{5}+\frac{2u^{3/2}}{3}+C$$

$$= \frac{2}{7}(1+x)^{7/2}-\frac{4}{5}(1+x)^{5/2}+\frac{2}{3}(1+x)^{3/2}+C$$

6-3 4B) $f(x) = \int [\csc(\sin x)]^2 \cos x \, dx$

Let $u = \sin x$

$du = \cos x \, dx$

$\int \csc^2(u) \, du$

$= -\cot(u) + C$

$$\int \csc^2(\sin x) \cos x \, dx = -\cot(\sin x) + C$$

6D) $\int \frac{dx}{\sqrt{x}(1+x)}$

Let $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2} x^{-1/2} dx$

$2du = \frac{1}{\sqrt{x}} dx$

$\Rightarrow 2 \int \frac{1}{1+u^2} du$

$= 2 \tan^{-1}(u) + C$

$= 2 \tan^{-1}(\sqrt{x}) + C$